

The loss-cone problem in dense nuclei

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ABSTRACT

We address the classical problem of star accretion onto a supermassive central gaseous object in a galactic nucleus. The resulting supermassive central gas-star object is assumed to be located at the centre of a dense stellar system for which we use a simplified model consisting of a Plummer model with an embedded density cusp using stellar point masses. From the number of stars belonging to the loss-cone, which plunge onto the central object on elongated orbits from outside, we estimate the accretion rate taking into account a possible anisotropy of the surrounding stellar distribution. The total heating rate in the supermassive star due to the loss-cone stars plunging onto it is estimated. This semi-analytical study, revisiting and expanding classical paper's work, is a starting point of future work on a more detailed study of early evolutionary phases of galactic nuclei. It merits closer examination, because it is one of the key features for the link between cosmology and galaxy formation.

Key words: galactic nuclei - star distribution - angular momentum - galactic structure - velocity distribution

1 GENERAL INTRODUCTION

Supermassive black holes (SMBH hereafter) lurking in centres of dense galactic nuclei, accreting stars and gas, provide under certain conditions the most powerful sources of energy in our visible universe, the quasars. That rather exotic idea in early time (Salpeter 1964, Zel'dovich 1964, Lynden-Bell 1969, Lynden-Bell & Rees 1971, Rees 1984, Begelman, Blandford & Rees 1984) has become common sense nowadays. Not only our own galaxy harbours a few million-solar mass black hole (Genzel et al. 2000, Genzel 2001, Ghez et al. 2000) but also many of other non-active galaxies show kinematic and gasdynamic evidence of these objects (Magorrian et al. 1998). The question how the black hole continues to grow, how it influences the stellar and gas distribution around itself, was intensively studied more than two decades ago (Frank & Rees 1976, Bahcall & Wolf 1976, Marchant & Shapiro 1980). Since it became clear relatively early that most supermassive black holes cannot be formed fast enough from stellar mass seed black holes in nuclei (Duncan & Shapiro 1982, 1983, but see also Lee (1995) for a somewhat differing view), they must be formed during the galaxy formation process directly, which is linked to cosmological boundary conditions. Rees (1984, 1996) argued that galactic nuclei in their formation process inevitably produce a dense core consisting of a star-gas system or a cluster of compact stellar evolution remnants, both ultimately collapsing to a supermassive black hole. Supermassive stars (SMS from now onwards) as such have been studied (Hoyle et al.

1964, Fricke 1973, 1974, Fuller et al. 1986) or dense supermassive star-gas composite objects as a transient progenitor of a SMBH (Hara 1978, Hagio 1986, Fuller et al. 1986, Langbein et al. 1990). Finally, the stability of compact dense star clusters was examined (Zel'dovich & Podurets 1965, Quinlan & Shapiro 1990). All these papers as a common feature conclude that, provided the central object, star cluster, SMS or a mixture of both, becomes smaller than a certain critical radius, it is able to undergo catastrophic collapse in a dynamical time scale due to an instability caused by Post-Newtonian relativistic corrections of hydrostatic equilibrium. The question, however, whether and how that final unstable state can be reached, is much less clear. Angular momentum of the protogalaxy or its dark matter halo, self-enrichment during the dissipative collapse providing opacity through lines which prevents collapse as compared to radiation driven expansion, and star-gas interactions heating the central massive gas object could all at least for some time prevent the ultimate collapse. Given the complex physical nature of the interstellar matter, star formation, and stellar interactions alone this is a complicated question and the conditions under which a supermassive object can form in a spherical, isolated star-forming and collapsing gas cloud, rotating or not, has to our knowledge never been exhaustively studied and answered (see however, some pioneering approaches such as Colgate 1967, Sanders 1970, Spitzer & Stone 1967, Spitzer & Saslaw 1966, Langbein et al. 1990, Quinlan & Shapiro 1990). The question has gained even more complexity, since we now know that the baryonic mat-

ter of galaxies collapses in their dominating dark halo, and that most galaxies and their dark halos experience merging with other dark haloes and large and small galaxies during the hierarchical gravitational structure formation (cf. e.g. Kauffmann et al. 1999a, b, Diaferio et al. 1999). This has led to another type of study of black hole statistics: due to e.g. Kormendy & Richstone (1995) and Magorrian et al. (1998) the black hole masses are well correlated with the bulge masses of their mother galaxies, and Gebhardt et al. (2000) find a correlation with the host galaxy velocity dispersion. These correlations support the idea that black hole formation is linked to galaxy formation. Such idea has been studied earlier, using a Press-Schechter hierarchical structure formation model (Efsthathiou & Rees 1988), and the results can be checked against the quasar luminosity function (Small & Blandford 1992, Eisenstein & Loeb 1995, Haehnelt & Rees 1993, Haehnelt et al. 1998, Boyle et al. 2000). On the basis of a statistical analysis using semi-analytical merger trees of galaxy building in a hierarchical structure formation picture Haehnelt & Kauffmann (2000) are able to reproduce the observed correlations using simple scaling relations for many of the in detail unknown physical processes, such as star formation, baryonic matter collapse in the halos, to mention only two examples. The detailed physics and parameters, how these processes work in a self-consistent model of black hole formation, however, are much less understood. So we lack any idea, what are the signatures of the black hole formation process in the morphology and kinematics of the innermost core and cusp regions, and to what extent they survive the merging history. Brave attempts to advance modelling in that domain (Rauch 1999) demonstrate in our view more the problems which still prevail originating from the large dynamical range of the problem and the complexity of the treatment of relaxation in a stellar system, rather than that they provide much reliable new insight.

Galaxy merging poses another serious problem due to the possibility that it can lead to two or more black holes in one nucleus, and the structure and kinematics is critically dependent on the evolution and possible gravitational radiation merger of the resulting black hole binary. For that problem direct N -body modelling is the only available possibility here using special GRAPE supercomputers (Makino & Ebisuzaki 1996, Makino 1997), general purpose supercomputers (Merritt, Cruz & Milosavljevic 2000), or a suitable hybrid method between direct and approximate N -body codes (Hemsendorf, Sigurdsson & Spurzem 2001). From these models it is yet unclear how fast in the real system dynamical friction, stochastic three-body interactions and external perturbations work together to produce eventually a single black hole again.

On the other hand, due to the ever increasing observational capabilities with ground and space based telescopes we get more and more detailed dynamical and photometric data of the structure of stellar systems around black holes. Therefore, we find it worthwhile to reconsider with present day numerical possibilities and increased knowledge about galaxy formation and evolution a detailed study of the evolution of dense star clusters, with gas, forming an SMS and its further evolution

In this paper we first reconsider the problem of a SMS in a dense star cluster and study with semi-analytic means its growth by star-gas interaction and trapping of stars, using

a fixed, approximate density and velocity dispersion profile of the surrounding star cluster. The size of the loss-cone and energy generation rate due to star-gas interactions are derived in a generalization of the “classical” determinations by Frank & Rees (1976), da Costa (1981) and Hara (1978). Subsequent work will incorporate these results in time-dependent numerical chemodynamical models of nuclei, first in spherical symmetry and later also for other non-symmetric configurations.

2 INTRODUCTION: THE LOSS-CONE, A REVIEW AND FURTHER DETAILS

Since the SMS scheme embraces the black hole accretion problem, we will have first a look at this problem in order to extract from it the global concept for the SMS. Thus, in this section we discuss the consequences from the astrophysical and dynamical point of view of the presence of a black hole (or a massive compact central object, from now onwards just BH) in a dense stellar object in about a relaxation time. We consider the steady-state distribution and consumption of stars orbiting a massive object at the centre of a spherical, stellar system.

The distribution of stars is determined by the relaxation processes associated with gravitational stellar encounters and by the consumption of low angular momentum stars which pass within a small distance of the central mass. Stars whose orbits carry them within the tidal radius r_t of the BH will be tidally disrupted. The *peribarthron* (distance of closest approach to the BH) is determined by the specific orbital angular momentum L and by the BH mass. There are situations in which the stars that have a radially elongated orbit and a low angular momentum pass close by the system centre and interact with the massive central object. In such a situation it is interesting to evaluate the density of those stars whose angular momenta are limited by a superior L_{\min} ; that is, the stars which belong to a defined region in the velocity-space.

Stars at a position r whose velocities are limited by a superior limit $v_{lc}(r)$ and, consequently, with an angular momentum $L < L_{\min} = rv_{lc}$, have orbits that will cross the tidal radius of the central BH in their motion. They will be disrupted due to the tidal forces and then they are lost for the stellar system. Such stars are said to belong to the *loss-cone*, since they are lost for the stellar system. The loss-cone is depleted in a crossing time $t_{\text{cross}} = r/\sigma$, where σ is the 1D velocity dispersion.

The diffusion of stars into this loss-cone has been studied by Frank and Rees (1976) and by Lightman and Shapiro (1977). We will talk about a “critical radius” within which stars on orbits with $r \leq r_{\text{crit}}$ diffuse. Inside they are swallowed by the BH, after being scattered to low angular momentum *loss-cone* orbits.

If there is a central point mass \mathcal{M}_\bullet , such that $\mathcal{M}_\bullet \gg m_*$, then its potential well will affect the stellar velocity field out to a distance

$$r_h = G\mathcal{M}_\bullet/\sigma^2. \quad (1)$$

This expression gives us the influence radius of the central object. G is the gravitational constant. The star gets disrupted whenever the work exerted over the star by the tidal

force exceeds its own binding energy. If we compute the work exerted over the star by the BH we can get an expression for the tidal radius,

$$r_t = \left[\frac{2}{3}(5-n) \frac{\mathcal{M}_\bullet}{m_*} \right]^{1/3} r_*, \quad (2)$$

where n is the polytropic index of the star, m_* the mass of the star and r_* its radius.

For solar-type stars it is (considering a $n = 3$ polytrope)

$$r_t \simeq 4.5 \cdot 10^{-8} \left(\frac{\mathcal{M}_\bullet}{M_\odot} \right)^{1/3} \text{ pc}. \quad (3)$$

2.1 Loss-cone phenomena

Frank and Rees (1976) studied how a stationary stellar density profile around a massive star accreting BH looks like. They found that the density profile follows a power-law within the region where the gravity of the massive star dominates the self-gravity of the stars,

$$\rho \propto r^{-7/4}. \quad (4)$$

This was followed by intensive numerical studies by other authors (cf. e.g. Shapiro & Marchant 1978, Marchant & Shapiro 1979, 1980 and Shapiro 1985) which are all in agreement with the first work of Frank and Rees.

The replenishment of the loss-cone happens thanks to the small-angle gravitational encounters in a timescale which is for the most real stellar systems slower than the dynamical processes.

We have loss-cone effects also in the neighbourhood of a massive central gas-formed object, an SMS: Stars with such orbits enter the gas-formed central object and lose kinetic energy if their density is high enough. Nevertheless, such stars will not disappear from the stellar system just by one crossing of the central object, as it happens for the BH problem. In this scenario the stars lose their energy in each crossing and their orbits come closer and closer to the central object until they are “trapped” in it, their orbits do not extend further than the massive central object radius (confined stars). This process was described qualitatively by Hara (1978), da Costa (1981) and Hagio (1986). Da Costa suggested the name “dissipation-cone”. However, we will keep the loss-cone term for it is the most commonly found in the related literature. We have to take into account that the meaning is that of a defined region of the velocity space at the position r , even though there is no quick loss.

Frank and Rees (1976) derived the following expression for the diffusion angle (the mean deviation of a star orbit in a dynamical time t_{dyn}):

$$\theta_D \simeq \sqrt{\frac{t_{\text{dyn}}}{t_{\text{relax}}}}. \quad (5)$$

where t_{dyn} and t_{relax} are the dynamical and relaxation times; namely,

$$t_{\text{dyn}} = r/\sigma_r(r) \quad (6)$$

$$t_{\text{relax}} = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m \rho(r) \ln(\eta N)}. \quad (7)$$

In this last expression (Larson 1970), ρ is the mean stellar mass density, $N = \frac{4}{3}\pi n_c r_c^3$ is the total particle number and η is a parameter of order unity which is set to be 0.4 (Spitzer 1958) or 0.11 (Giersz and Heggie 1994); its exact value cannot be defined easily and depends on the initial model and the anisotropy. Here we use $\eta = 0.11$.

Now we look for a condition at a place $r > r_h$ for a star to touch or to cross the influence radius of the central object within a crossing time. For this aim we look now for the amount of stars which reaches the central influence radius with an unperturbed orbit. Unperturbed here means that the star orbit results from the influence of the gravitational potential and from that of the rest of the stars and of the central object, and it is not affected by the local, two-body, small- or big-angle gravitational encounters. We envisage then the average part of the gravitational potential, whereas the random component due to the individual behaviour of the stars will be neglected.

To define the loss-cone angle we say that a star belongs to this cone when its distance to the peribarthron (which depends on the orbit we have, i.e., on the energy E and angular momentum) is less or equal to the tidal radius, $r_p(E, L) \leq r_t$, $\theta \leq \theta_{lc}$.

There is a maximum θ for which the peribarthron radius is less than or equal to the tidal radius. We define this as θ_{lc} (where the subscript “lc” stands for loss-cone).

We derive the $v_{lc}(r)$ using angular momentum and energy conservation arguments. We just have to evaluate it at a general radius r and at the tidal radius r_t , where the tangential velocity is maximal and the radial velocity cancels:

$$v_{lc}(r) = \frac{r_t}{\sqrt{r^2 - r_t^2}} \cdot \sqrt{2[\phi(r_t) - \phi(r)] + v_r(r)^2}. \quad (8)$$

2.2 The critical radius

It is interesting to evaluate a certain radius which Frank and Rees (1976) introduced by defining the ratio $\xi := \theta_{lc}/\theta_D$. When $\xi = 1$, then $\theta_{lc} = \theta_D$, and this corresponds to a “critical radius”, r_{crit} , if there is only one radius with this condition. Inside the critical radius (i.e. $\xi > 1$, $\theta_{lc} > \theta_D$) stars are removed on a t_{dyn} . For larger radii (i.e. $\xi < 1$, $\theta_{lc} < \theta_D$) we cannot talk about a “loss-cone” because this θ_D corresponds to the variation of θ within a t_{dyn} , and this is the required time for the star within the loss-cone to plunge onto the BH if the orbit is unperturbed. The angle variation happens sooner than the required time for stars to sink into the loss-cone. If $\theta_D > \theta_{lc}$, loss-cone stars can get in and out of the loss-cone faster than they could reach the central object.

3 THE LOSS-CONE CONTRIBUTION TO THE HEATING RATE OF THE GAS

3.1 Introduction

Dissipation of the stellar kinetic energy of a star plunging onto the SMS and suffering from the drag force leads to a heating of the SMS. Another possible consequence of the local star-gas interaction is the formation of massive stars within the cloud due to the accretion of ambient gas (Da Costa 1979). This could increase the supernova rate and

be an important source of energy. Here we will assume an equilibrium for the SMS for the timescales of interest.

The star distribution will be affected at large radii ($r \gg \mathcal{R}_s$, \mathcal{R}_s being the SMS radius) by removal of stars in the central regions of a stellar system (Peebles, 1972). A drift of stars occurs in the centre of the stellar system in order to recover the equilibrium. In the special case of having a BH at the centre of the system, processes like tidal disruption lead to the destruction of the star. In this arena we have an outward energy flux created by the sinking of stars via relaxation processes (local, two-body, small-angle gravitational encounters).

When we consider the general case of an SMS, the basic picture is the same, but some aspects vary; the removal of stars and the inward transport are due to a different process. The inward flux of energy is produced not only for the local, two-body, small-angle gravitational encounters, but also for dissipative processes. The effective sinking for stars is now related to gas-drag energy dissipation and can involve or not their actual physical destruction. A star moving through the cloud will quickly dissipate energy because of gas drag and then it will sink into the centre of the SMS. The main difference between a BH and an SMS is basically that the former produces a low-angular momentum star depletion on a crossing time, whereas the dense gas cloud or SMS does the same but in a dissipation time.

Since we want to analyse the effects on the dense stellar system arising from the presence of a central gas cloud we have to distinguish between those stars whose orbits are limited to the region where the SMS is located (*confined stars*) and those stars in orbits which surpass the radius of the cloud (*unconfined stars*).

When we talk about confined stars, the first steps in the evolution are determined by the energy dissipation given by the drag force that the dense gas cloud exerts on the individual stars. Stars lose velocity in their motion inside the cloud, they are slowed down by the gas and therefore they cede heat to the cloud.

The slowing down of the stars makes them become a more compact subsystem which will sink down to the centre of the cloud. The system becomes self-gravitating and we have a cusp in the stellar distribution. However, star-star interactions can play a decisive role in this point, since they yield a depletion in the number of confined stars, or we can have direct collisions between them and thus disruption or coalescence. This could avoid that a singularity in the core collapse crops up. We cannot exclude the exchange of mass between individual stars and the gas as another possible way to prevent the singularity, since this can yield the star disruption via stellar wind, or the creation of heavier stars which become a supernova (Da Costa 1979).

The consequent evolution of the confined system will be in part determined by the rate at which surrounding stars outside the gas cloud refill this confined-stars gap. The importance of the core collapse will also be a decisive point for the evolution.

Unconfined stars move on orbits extending larger than the SMS radius. However, they can suffer its influence out to a radius within which the presence of the SMS is effective. The idea is exactly the same as for the BH, for the SMS is a generalisation of the former case

3.2 Kinetic energy dissipation

The drag force that the individual stars suffer when they cross the SMS is given by the next equation estimated by Bisnovat'i-Kogan and S'un'aev (1972):

$$F_D = C_D S \rho_{sms} v_*^2. \quad (9)$$

C_D is a numerical parameter of order unity, ρ_{sms} is the mean density of the gas cloud, v_* is the velocity of the stars and S is the cross section of the stars, $S = \pi r_*^2$. In case of a supersonic motion of the star the force F_D can be interpreted as caused by the ram pressure (pressure difference) originating at a bow shock in front of the moving star. Due to the physical shock conditions one can show in such a case $C_D \approx 4$ (Courant & Friedrichs 1998).

Suppose that the star crosses the SMS from one extreme to the opposite, i.e. along its diameter; thus, if \mathcal{R}_s is the radius of the SMS, the stellar energy dissipated during each passage through it is

$$\Delta E_D = F_D \cdot 2\mathcal{R}_s. \quad (10)$$

The orbits of the stars within r_h will be elliptic shaped with one focus at the SMS centre. The semi-major orbit axis a will shrink because of the drag force, driving the orbit directly into the SMS. The average energy dissipation rate is

$$-\frac{dE}{dt} = \frac{\Delta E_D}{T} = \frac{2C_D \rho_{sms} \pi r_*^2 G \mathcal{M}_s}{\pi \sqrt{4a^3 / G \mathcal{M}_s}}, \quad (11)$$

where T is the period and \mathcal{M}_s the mass of the SMS.

3.3 Loss-cone stars velocity field distribution function

It stands to reason that at distances much larger than the SMS radius ($r \gg \mathcal{R}_s$) the star field velocity distribution has a Maxwellian shape:

$$f(v_t, v_r) = \frac{\rho(r)}{(2\pi)^{3/2} \sigma_r \sigma_t^2} \exp\left(-\frac{v_r^2}{2\sigma_r^2}\right) \exp\left(-\frac{v_t^2}{2\sigma_t^2}\right). \quad (12)$$

In order to get the density of stars within the loss-cone we have to compute the following integral:

$$\rho_{lc}(r) = \int_{-v_r|_{\max}}^0 \int_{-v_{lc}}^{+v_{lc}} f(v_r, v_\theta, v_\phi) dv_r dv_\theta dv_\phi. \quad (13)$$

Taking into account that $dv_\theta dv_\phi = 2\pi v_t dv_t$ and that $f(r, v) = f(r, -v)$,

$$\rho_{lc}(r) = 4\pi \rho(r) \int_0^{v_r|_{\max}} dv_r \int_0^{v_{lc}(r, v_r)} f(r, v_r, v_t) v_t dv_t. \quad (14)$$

In this expression, the maximal radial velocity is $v_r|_{\max} = v_{\text{escape}} = \sqrt{2\phi(r)}$ and the potential is the sum of both, the supermassive star and the stellar system potential,

$$\phi(r) = \phi_{sms}(r) + \phi_*(r). \quad (15)$$

The integral happens to be analytical, and it yields the following result:

$$\rho_{lc}(r, v_r) = \rho(r) \cdot \{\alpha - \zeta \cdot \beta \cdot \psi\}, \quad (16)$$

where

$$\alpha \equiv \text{erf}(\sqrt{\phi(r)/\sigma_r^2})$$

$$\beta \equiv \exp\left(-\frac{\mathcal{R}_s^2}{r^2 - \mathcal{R}_s^2} \frac{2\Delta\phi}{\sigma_t^2}\right)$$

$$\psi \equiv \operatorname{erf}\left(\frac{1}{\zeta} \sqrt{\phi(r)/\sigma_r^2}\right)$$

$$\zeta \equiv \left[\frac{(r^2 - \mathcal{R}_s^2)\sigma_t^2}{(r^2 - \mathcal{R}_s^2)\sigma_t^2 + \mathcal{R}_s^2\sigma_r^2}\right]^{1/2}$$

$$\Delta\phi \equiv \phi(\mathcal{R}_s) - \phi(r).$$

Since we are working with a Gaussian function whose width is σ , the contributions of velocities $v_r > 2\sigma_r$ to the total mass are small and therefore negligible. In the practice it means that we can approximate the integral by $v_r = 2\sigma_r$. In such a situation, the loss-cone adopts an easy geometrical form: an open cylinder in the $-v_r$ direction with a radius v_{lc} . The resulting integral yields

$$\frac{\rho_{lc}(r)}{\rho(r)} = \left[1 - \exp\left\{-\frac{\mathcal{R}_s^2}{r^2 - \mathcal{R}_s^2} \frac{2\sigma_r^2 + \Delta\phi(r)}{\sigma_t^2}\right\}\right] \times \operatorname{erf}\left(\sqrt{\frac{\phi(r)}{\sigma_r^2}}\right) \quad (17)$$

We use for the stellar system a Plummer model. Thus,

$$\phi(r) = \frac{GM_s}{r} + \frac{GM_*}{\sqrt{r^2 + r_c^2}}. \quad (18)$$

Therewith, the resulting expression is

$$\frac{\rho_{lc}(r)}{\rho(r)} = (1 - \exp A) \cdot \operatorname{erf}(B), \quad (19)$$

where

$$A \equiv -\frac{\mathcal{R}_s^2}{r^2 - \mathcal{R}_s^2} \frac{1}{\sigma_t^2} (C)$$

$$B \equiv \frac{1}{\sigma_r} \sqrt{\frac{GM_s}{r} + \frac{GM_*}{\sqrt{r^2 + r_c^2}}}$$

$$C \equiv 2\sigma_r^2 + \frac{GM_s}{\mathcal{R}_s} - \frac{GM_s}{r} + \frac{GM_*}{\sqrt{\mathcal{R}_s^2 + r_c^2}} - \frac{GM_*}{\sqrt{r^2 + r_c^2}}.$$

The velocity vectors of all stars which belong to a given (fixed) phase space density $f = f_0 = \text{const}$ shape an ellipsoid whose two tangential and one radial major axes have lengths equal to the velocity dispersion: $\sigma_\theta = \sigma_\phi$, and σ_r (Frank & Rees 1976). If one uses $f_0 = f(\sigma_r, \sigma_\theta, \sigma_\phi)$, the surface of the ellipsoid $A = \pi\sigma_\theta\sigma_\phi\sigma_r$ is a measure for the available velocity space.

A cone of angle θ_{lc} ,

$$\theta_{lc} := \arcsin\left(\frac{v_{lc}}{\sigma_r}\right), \quad (20)$$

cuts out a segment of the foregoing velocity ellipsoid's A surface of corresponding fraction surface A_{lc} ,

$$A_{lc} \approx \pi\theta_{lc}^2, \quad (\theta_{lc} \ll \pi). \quad (21)$$

We can then define the ratio $\Omega' := A_{lc}/A \approx \theta_{lc}^2/4$, which is a measure for the loss-cone size.

Spurzem (1988) proofs that, with the assumption that v_{lc} does not depend on v_r any more and taking into account that with a Schwarzschild-Boltzmann distribution we can reduce the loss-cone momenta to elementary Gaussian error functions, the quantity

$$\Omega := \frac{\rho_{lc}}{\rho} \quad (22)$$

yields in first order

$$\Omega \approx \frac{v_{lc}^2}{4\sigma_\theta^2} \approx \frac{\theta_{lc}^2}{4}. \quad (23)$$

This is the connection between the preceding simple picture of the loss-cone and the definition of Ω in the velocity space: For a Schwarzschild-Boltzmann distribution we can find out that, at first order, Ω is of the same size as Ω' .

3.4 Isotropy and anisotropy in the stellar system

We introduce now an *isotropy ratio* in order to study the different possible situations for the stellar distribution: The tangential velocity dispersion is $\sigma_t^2 = \sigma_\phi^2 + \sigma_\theta^2$; in case of isotropy, $\sigma_r^2 = \sigma_\phi^2 + \sigma_\theta^2$, then $\sigma_t^2 = 2\sigma_r^2$.

Now we define the ratio $R := 2\sigma_r^2/\sigma_t^2$. According to this definition, $R = 1$ for the isotropic case. The corresponding values of R for radial and tangential anisotropy can be obtained bearing in mind that $\sigma^2 = \sigma_r^2 + \sigma_t^2 = \sigma_t^2(R/2 + 1)$, $\sigma_t = \sigma/\sqrt{R/2 + 1}$ and $\sigma_r = \sigma_t \cdot \sqrt{R/2}$.

We have the loss-cone star density as a function of the supermassive star radius, \mathcal{R}_s . This does not provide much information, since in principle this radius could have any size; we do not have a criterion for it yet. Instead, what does make sense is to express this loss-cone star density in terms of the supermassive star stability, which is something has been studied in detail (Fuller & Woosley, 1986).

From Chandrasekhar (1964), instability sets in when the radius of the star \mathcal{R}_s is less than a critical radius $\mathcal{R}_s^{\text{crit}}$. He shows that if the ratio of specific heats $\gamma = C_p/C_v$ exceeds 4/3 only by a small amount, then dynamical instability will occur if the mass contract to the radius $\mathcal{R}_s^{\text{crit}}$

$$\mathcal{R}_s^{\text{crit}} = \frac{K}{\gamma - \frac{4}{3}} \left(\frac{2GM_s}{c^2} \right). \quad (24)$$

Thus, we introduce the *stability coefficient* $\delta := \mathcal{R}_s/\mathcal{R}_s^{\text{crit}}$. We just have to substitute $\mathcal{R}_s = \delta \cdot \mathcal{R}_s^{\text{crit}}$ in the loss-cone star density formula and vary δ instead of \mathcal{R}_s .

3.5 Connection at the influence radius

Since we are interested in the diffusion angle, we now derive two expressions for it, within the influence radius of the SMS and outside it. For this aim we look at the dynamical and relaxation time at this radius.

Within the influence radius the velocity dispersion is $\sigma(r) = \sqrt{GM_s/r}$. This is just an approximation which we make here for simplicity, since our radii are $r < r_h$. To include $r \ll r_h$ we need a better model, which can only be obtained by numerical solution of the equation of Poisson. Outside the influence radius, we use a modified Plummer model for the velocity dispersion, since we have to match both solutions, within and outside the SMS influence radius, since we have to look for a velocity dispersion connection; otherwise we get artificial, non-physical “jumps” in the plots for $r \approx r_h$. This can be performed by adding a factor α to the Plummer velocity dispersion expression, which we determine by requiring both velocities dispersions to be equal at the influence radius, $\sigma(r_h)|_{r < r_h} \equiv \sigma(r_h)|_{r > r_h}$. Thus,

$$\sqrt{\frac{GM_s}{r_h}} = \alpha \cdot \sqrt{\frac{GM_*}{6r_c}} \left(1 + \frac{r_h^2}{r_c^2}\right)^{-1/4}. \quad (25)$$

Therefore, the velocity dispersion outside the influence radius is:

$$\sigma(r) = \sqrt{\frac{GM_s}{r_c r_h}} \cdot \left[\frac{r_c^2}{r_c^2 + r^2}(r_c^2 + r_h^2)\right]^{1/4}. \quad (26)$$

Note that α is necessary because our velocity dispersion is approximate for $r < r_h$, but not for $r \ll r_h$.

For the dynamical and relaxation times we can get their corresponding expressions thanks to expressions (6) and (7).

4 MASS ACCRETION RATES

The rate of stars plunging onto the central SMS is given by two different formulae, depending on whether or not there is a crossing point for the θ_{lc} , θ_D -plot against the radius, *a critical radius*. If we find that the curves happen to cross, the mass accretion rate \dot{M} has the expression $\dot{M} = M_*(r_h)/t_{\text{relax}}(r_h)$ because the loss-cone will be depleted in a relaxation time, and the mass to take into account is that which lays within the critical radius. On the other hand, if there is no crossing point, this means that the loss-cone is not empty and in this situation we have to employ a rather different expression, we have to resort to the loss-cone star density expression to get the mass being accreted into the SMS. In this case the timescale of interest is the dynamical time, $\dot{M} = \Omega(r)M_*(r)/t_{\text{dyn}}(r)$.

It may be asserted, nevertheless, that this formula is not completely correct because it is based on the stationary model, which supposes an empty loss-cone in the first case and a full loss-cone in the second case. We have to generalise it by means of a “diffusion” model (Spurzem 2000). We introduce the concept of the *filling degree* k of the loss-cone as follows: Let us conjecture that f is the unperturbed velocity distribution. If the loss-cone is empty and angular momentum diffusion is neglected, then $f = 0$ inside the loss-cone and f remains unchanged elsewhere in velocity space. Actually, this distribution function will have a continuous transition from nearly unperturbed values at large angular momenta towards a partially depleted value inside the loss-cone. We approximate this by a distribution function f having a sudden jump just at the value $L_{\text{min}} = m_* v_{lc}$ from an unperturbed value f_0 , $f = k \cdot f_0$, with $0 \leq k \leq 1$. Since we work with the hypothesis that some kind of stationary state is to be established in the limit $t \rightarrow \infty$, the filling degree is

$$k_\infty = \frac{\nu(1+\nu)}{1+\nu(1+\nu)}. \quad (27)$$

In this expression $\nu \equiv \theta_D^2/\theta_{lc}^2$. Then we have to multiply the accretion rates by this filling degree k_∞ .

The stellar mass within and outside the influence radius is

$$M(r, r_{\text{min}})|_{r < r_h} = \frac{16\pi}{5} \rho(r_h) r_h^3 \left(\frac{r}{r_h}\right)^{5/4}, \quad (28)$$

$$M(r)|_{r > r_h} = M(r_h)|_{r < r_h} + 4\pi \int_{r_h}^r \rho(r') r'^2 dr' \quad (29)$$

$$= \frac{16\pi}{5} \rho(r_h) r_h^3 + 4\pi \frac{M_*}{r_c^3} \left[\frac{r^3}{(1+r^2/r_c^2)^{3/2}} - \frac{r_h^3}{(1+r_h^2/r_c^2)^{3/2}} \right].$$

To get the total heating rates, we just have to compute the value of

$$L_{\text{heat, all}} = \left(\frac{\dot{M}}{m_*}\right) \cdot E_{\text{heat, 1*}}, \quad (30)$$

where $E_{\text{heat, 1*}}$ is the heating for one star (during one crossing).

5 PLOTS AND RESULTS

In this section we analyse the interaction rate of stars with the SMS by varying the parameters introduced in the former sections, namely δ , the core radius r_c , the total stellar mass and the supermassive star mass itself. We suppose that the stars which conform the stellar system are solar-type stars. For all the plots we extend the radii down to 1.001 times the SMS radius, because we would run into a snag if we extended it within the SMS radius, since the proportions $\sigma^2 \propto 1/r$ and $\rho \propto r^{-7/4}$ would be wrong and the loss-cone star density and therefore the loss-cone angle has been obtained considering a Maxwell-Boltzmann distribution. It would also be an error of the problem conception itself, because we are studying the non-confined stars and the loss-cone, and its definition does not make any sense for radii less than the \mathcal{R}_s . This explains the first inequality in $\mathcal{R}_s < r_h < r_c$, which is an exigency that we must follow unflinching because, otherwise, the Plummer model, which we use for the stellar system, would not be a suitable solution for it in the case that $r_h < r_c$ is not satisfied. However, what we demand here is not a requirement of the physics of the problem, but a condition for the method being employed to solve it. Situations in which $r_h < \mathcal{R}_s$ or $r_h > r_c$ have to be solved numerically for the equation of Poisson and the velocity distributions.

In order to estimate the heating rates for a single star crossing the SMS we have to plot out the mass accretion rate of this central massive object in the case that we have no crossing point for the loss-cone and diffusion angle curves, whereas if we have a critical radius we will have to compute the total stellar mass and dynamical time at this value.

In Fig. 1 we plot the velocity dispersion against the radius for a $10^3 M_\odot$ SMS. We observe a typical power law of $\propto r^{-1/2}$ within the influence radius -which we represent by a vertical dashed line for all cases- because we have a cusp on velocities in this interval of radii. We have a nearly constant velocity for later radii which lie in the section of values close to the core radius. Then the velocity dispersion decays. The length of this nearly constant velocity section as well as the slope for the decay depends on the galactic nuclei in the galaxy.

Regarding Fig. 2, we show the loss-cone normalised density difference between the isotropic and radially anisotropic cases for a $10^7 M_\odot$ SMS. In this plot we can observe a bigger number of stars being accreted into the SMS for the radially anisotropic case, since a radial orbit means a lower angular momentum and thus it is more probable that the star sinks into the central object, and vice-versa for a tangential orbit (Fig. 3).

As regards the loss-cone and diffusion angle plots, we

examine two different cases in Fig. 3 and 4: a $10^4 M_\odot$ and a $10^7 M_\odot$ SMS. One may observe that for the first one we find a critical radius, whereas for the latter one the curves do not intersect. It is also interesting to find out which angle is bigger and where, since for a $\theta_{lc} \gg \theta_D$ we have an almost empty loss-cone, because it is quickly depleted and the stars replenish it very slowly; the opposite case, $\theta_{lc} \ll \theta_D$, implies that the loss-cone is full.

We get a maximum \dot{M} at about a core radius in the mass accretion rates plot for a $10^7 M_\odot$ SMS, because the biggest contribution of stars being accreted into the SMS lies at this radius. Figure 5 shows an irregularity at the influence radius, because the loss-cone and diffusion angle plots show us that the former happens to be always bigger than this one and thus we have to apply the approximation commented in the foregoing section.

5.1 Heating rates. An estimation

When we look at the \dot{M} for the $10^3 M_\odot$ and $10^4 M_\odot$ SMS's, we obtain that for the $10^3 M_\odot$: $\dot{M}_{\text{iso}} = 1.75 \times 10^{-13} M_\odot/\text{yr}$, $\dot{M}_{\text{tan}} = 1.66 \times 10^{-13} M_\odot/\text{yr}$, $\dot{M}_{\text{rad}} = 1.97 \times 10^{-13} M_\odot/\text{yr}$, where the subscript “iso” stands for isotropy, “rad” for radial anisotropy and “tan” for tangential anisotropy.

The case of $10^4 M_\odot$ is similar to the last one. For the $10^7 M_\odot$ SMS, we will select the \dot{M} corresponding to the core radius, since the most important contribution is reached there: $\dot{M}|_{\text{core}} = 10^{-2} M_\odot/\text{yr}$. To get the heating rates of these non-confined stars, we just have to compute

$$\dot{E} = \left(\frac{\dot{M}}{M_\odot} \right) \pi r_*^2 \rho_{\text{sms}} v_*^2 \cdot 2\mathcal{R}_s / t_{\text{cross}}, \quad (31)$$

where $v_*^2 = GM_s/\mathcal{R}_s$, $t_{\text{cross}} = 2\mathcal{R}_s/v_*$, $\rho_{\text{sms}} = \mathcal{M}_s/(\frac{4\pi}{3}\mathcal{R}_s^3)$. For the SMS we have supposed, as a first approximation, a constant density.

The corresponding \mathcal{R}_s are:

$$10^3 M_\odot: \mathcal{R}_s = 2.5 \times 10^{-9} \text{ pc}$$

$$10^4 M_\odot: \mathcal{R}_s = 8 \times 10^{-8} \text{ pc}$$

$$10^7 M_\odot: \mathcal{R}_s = 2.5 \times 10^{-2} \text{ pc}.$$

The luminosities are:

$$L_{10^3}/L_\odot = 6.2 \times 10^3$$

$$L_{10^4}/L_\odot = 1.17 \times 10^4$$

$$L_{10^7}/L_\odot = 5.6 \times 10^5,$$

where L_{10^i} stands for the $10^i M_\odot$ SMS luminosity. It is not a surprise that these luminosities are not sufficient to support quasar luminosities, which was known before. We confirm however the earlier result by Langbein et al. (1990) with our more detailed, but stationary loss-cone model, that the luminosities are large enough to prevent for some time the relativistic collapse of a SMS in a galactic centre; that was called the quasi-pile stage by Hara (1978).

6 CONCLUSION AND DISCUSSION

We have revisited the classical loss-cone semi-analytic theory invented by Frank & Rees (1976) for star accretion onto central supermassive black holes in galactic nuclei and star clusters and extended by da Costa (1981) for the case of stars on radial orbits being trapped by star-gas interactions

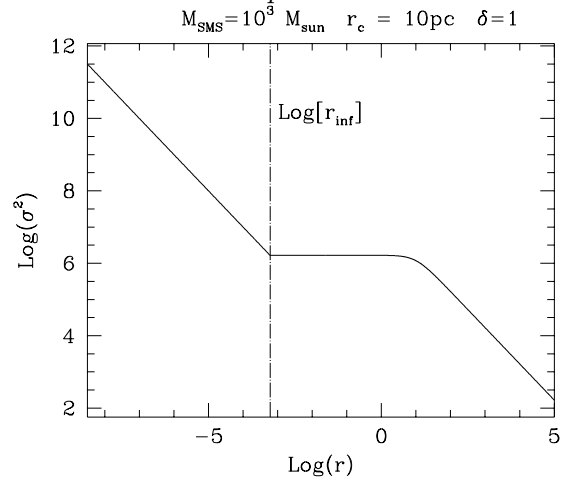


Figure 1. The velocity units are $(m/s)^2$ and r is expressed in pc. The influence radius is located at $6 \cdot 10^{-4}$ pc. We show its logarithm in the plot with a vertical dashed line. The central velocity dispersion (the velocity dispersion at the influence radius) is $\sigma_{\text{central}} = \sqrt{GM_*/(6r_c)} = 84 \text{ km/s}$. M_{SMS} stands for \mathcal{M}_s .

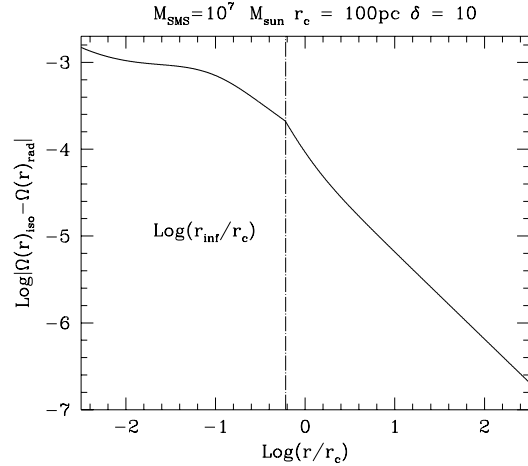


Figure 2. $\Omega(r) := \rho_{lc}(r)/\rho(r)$ decays with the distance to the SMS. We have divided the radius by the core radius in order to normalise it. A bigger number of stars plunge onto the SMS for the radially anisotropic case.

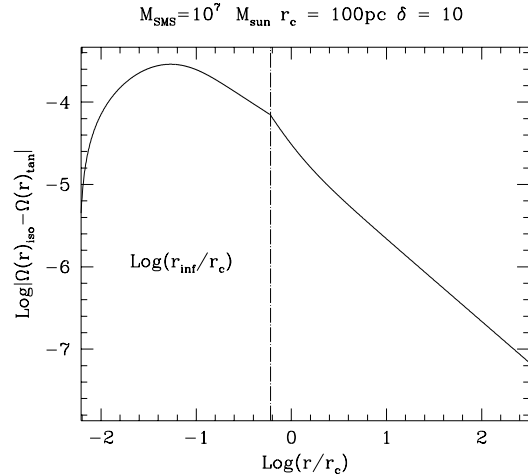


Figure 3. The same as for Fig.2 but for the tangentially anisotropic case and a lower number of stars.

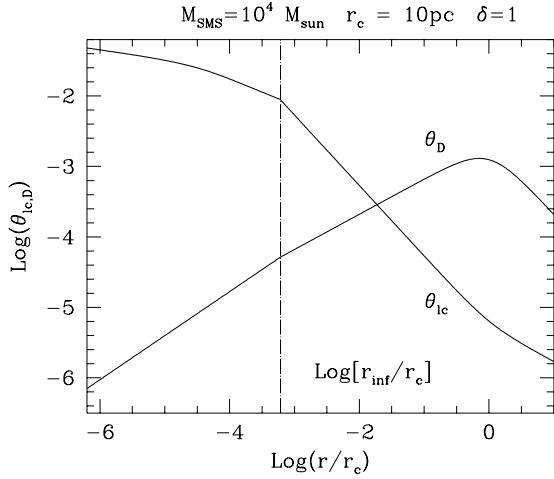


Figure 4. $\theta_{lc} = 2 \cdot \sqrt{\Omega(r)}$, $\theta_D = \sqrt{t_{\text{dyn}}/t_{\text{relax}}}$. For this mass the two curves cross at the critical radius. For radii smaller than that, the loss-cone angle is bigger than the diffusion angle, and this implies that the loss-cone is empty. From the r_{crit} onwards it is no longer empty, for $\theta_D > \theta_{lc}$. With the crossing point we can work out the accretion rate and find out the differences depending on whether we consider an isotropic situation for the stellar velocity distribution function or an anisotropic one, distinguishing between a radial or tangential anisotropy. If we set this case against the $10^3 M_\odot$ we do not find big differences.

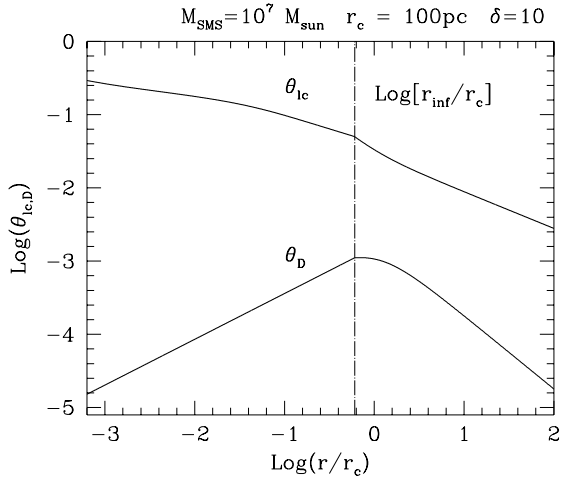


Figure 5. For higher masses, such as $10^7 M_\odot$, the loss-cone is empty. We get no critical radius.

in a central supermassive star-gas system. In Langbein et al. (1990) such model was included in time-dependent, spherically symmetric models of star-gas systems in galactic nuclei. Though highly idealized we think such configurations are still worth a study to understand the physical processes at work in the early formation phase of massive galaxies with formation of central black holes. Notions such as the critical radius in the classical work, where the loss-cone star accretion becomes important and flattens out the cusp density profile turn out naturally in our model without any ad hoc assumptions. While in this research note we keep the stellar background system fixed and develop the old ideas in an up-to-date form, we will present in subsequent papers their inclusion into a self-consistent dynamical model of relaxing star clusters with central black hole, first in spherical sym-

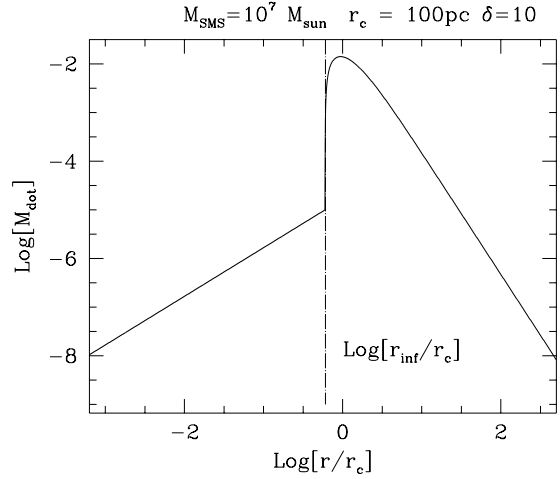


Figure 6. The “jump” is due to the approximation we made in order to get the accretion rates. The maximum is reached at a distance which corresponds to the core radius.

metry (using high resolution anisotropic gas models as described in Louis & Spurzem 1991, Spurzem 1994 and Giersz & Spurzem 1994), later going to more general symmetries or triaxial systems. Recent dramatic improvements of the observational situation and the demography of black holes in nuclei demand such progress in dynamical modelling, which is yet surprisingly poor (see e.g. Richstone 1998). It is clear that in a proper cosmological context many complications occur for simplified modelling: the whole system is embedded in a collisionless dark halo with disputed central density profile, it is non-stationary due to a sequence of merger events in hierarchical structure formation, with the possible formation of binary or multiple black holes and perturbations of various kinds will even cause a single black hole not to be fixed in the centre. Despite of all that have begun our work at the point where we think present (astro)physical understanding and modelling comes to its limits, and this is the case for a spherical dense large N star cluster, suffering from relaxation and star accretion around a fixed massive black hole.

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